2. Dada la matriz

$$A = \begin{pmatrix} -1 & 2 & 0 & 1 \\ 3 & -6 & 2 & 0 \\ -2 & 4 & -2 & -1 \end{pmatrix}$$

A) Calcular la forma normal de Hermite H de la matriz A y la matriz regular Q tal que QA = H. Deducir de lo anterior el rango de A

B) Si A es la matriz ampliada de un sistema AX = b, discutir y resolver, si es posible, usando Cramer.

A) $$(A \mid Id) = \left(\begin{array}{cccc|ccc} -1 & 2 & 0 & 1 & 1 & 0 & 0 \\ 3 & -6 & 2 & 0 & 0 & 1 & 0 \\ -2 & 4 & -2 & -1 & 0 & 0 & 1 \end{array}\right) \sim_f \begin{array}{l} F_1 \leftrightarrow -F_1 \\ F_2 \to F_2 - 3F_1 \\ F_3 \to F_3 + 2F_1 \end{array}$$

$$\sim_f \left(\begin{array}{ccccccc} 1 & -2 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 & 3 & 1 & 0 \\ 0 & 0 & -2 & -3 & -2 & 0 & 1 \end{array}\right) \sim_f \begin{array}{l} F_2 \leftrightarrow \frac{1}{2} F_2 \\ F_3 \leftrightarrow F_3 + 2F_2 \end{array}$$

$$\sim_f \left(\begin{array}{cccc|ccc} 1 & -2 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 3/2 & 3/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right)$$

(left block $= H$, right block $= Q$ regular)

$Q \cdot A = H$

$rg(A) = n^{\underline{o}}$ pivotes de $H = 2$

B)
$$\left.\begin{array}{r} -x + 2y = 1 \\ 3x - 6y + 2z = 0 \\ \cancel{-2x + 4y - 2z = -1} \end{array}\right\} \Rightarrow \left.\begin{array}{r} 2y = 1 + x \\ -6y + 2z = -3x \end{array}\right\}$$

$$\det\begin{pmatrix} -1 & 2 & 0 \\ 3 & -6 & 2 \\ -2 & 4 & -2 \end{pmatrix} = 0 \qquad \begin{vmatrix} 2 & 0 \\ -6 & 2 \end{vmatrix} = 4 \neq 0$$

$$\left.\begin{array}{r} 2y = 1+\lambda \\ -6y+2z = -3\lambda \end{array}\right\} \quad x=\lambda$$

$$y = \frac{\begin{vmatrix} 1+\lambda & 0 \\ -3\lambda & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 0 \\ -6 & 2 \end{vmatrix}} = \frac{2(1+\lambda)}{4} = \frac{1}{2}(1+\lambda)$$

$$z = \frac{\begin{vmatrix} 2 & 1+\lambda \\ -6 & -3\lambda \end{vmatrix}}{\begin{vmatrix} 2 & 0 \\ -6 & 0 \end{vmatrix}} = \frac{\cancel{-6\lambda}+6(1+\cancel{\lambda})}{4} = \frac{3}{2}$$

$$\boxed{\text{Solución: } \left(\lambda, \frac{1}{2}(1+\lambda), \frac{3}{2}\right)}$$